Technical Notes

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Thermal Instabilities of the Anode in a Magnetoplasmadynamic Thruster

V. V. Subramaniam*
Ohio State University, Columbus, Ohio
and
J. L. Lawless†
Space Power Inc., San Jose, California

Nomenclature

A = thermionic emission coefficient
d = anode thickness
e = electronic charge
J = total current
j = current density passing through the anode
j_e = electron current density from the plasma (Its sense is positive away from the anode.)
j_E = current density of thermionically emitted electrons (Its sense is positive into the anode.)
j_∞ = average net current density through the plasma, taken positive away from the anode
k = Boltzmann's constant
L = anode length
m_e = electron mass
n_e = electron number density at the plasma-sheath boundary
$O(\,)$ = order of magnitude of the quantity in parenthesis
q_0 = heat flux on the anode inner surface (Its sense is positive into the anode surface.)
T = temperature
T_0 = anode inner surface temperature
T_{0c} = critical value of T_0 where $\partial j_\infty/\partial T_0 = 0$
T_d = anode outer surface temperature
T_e = plasma electron temperature
V_A = anode sheath voltage drop, defined as the potential at the plasma-sheath edge minus the anode potential
W = anode width
x = outward coordinate taken to be 0 at the anode inner surface, and d at the anode outer surface
ϵ = emissivity of outer anode surface
λ = thermal conductivity of anode material
σ_{SB} = Stefan-Boltzmann constant
σ = electrical conductivity of anode material
ϕ_A = anode material work function

I. Introduction

An important consideration in the use of magnetoplasmadynamic (MPD) thrusters operating at steady state for space missions is electrode erosion. Erosion in MPD thrusters has been studied experimentally and has focused mainly on the cathode.[1-3] There are also ongoing experiments in cathode erosion.[4] However, in this article, we present a simple analysis of the energy balance on the anode in an effort to predict anode surface temperatures. This can subsequently be used to predict erosion rates by evaporation. As will be shown, even such a simple model reveals subtle physical phenomena.

Electrode erosion is connected with the modes of current conduction through the electrode surface. Two modes are known to exist: spot and diffuse. The diffuse mode at subonset conditions is the focus of this article. Vainberg et al.[5] have considered anode behavior at onset conditions and beyond. Their explanation of anode melting rests on the anode sheath reversal mechanism, which has also been observed by Hugel.[6] In constrast to these earlier works, it is shown in this article that a thermal runaway may occur well *before* the sheath reversal. This work supports earlier conjectures that local surface melting of the anode precedes spot formation.[7] Two operating modes are predicted by the theory presented here. One of these yields a stable steady-state temperature for the anode, whereas the other results in a thermal runaway in which the anode regeneratively heats itself until it melts. The total current, anode geometry, and material work function are shown to strongly influence the steady-state anode temperature for the stable operating regime.

In the following section, the governing equations describing anode heat transfer will be derived. The solutions to these equations under some conditions of interest are given and discussed in Sec. III, followed by the summary and conclusions in Sec. IV.

II. Anode Energy Balance

Consider the hollow cylindrical anode geometry of the MPD thruster modeled as a long thin slab of length L, width W, and thickness d (shown in Fig. 1). At steady state, the energy balance gives

$$\lambda\frac{d^2T}{dx^2} = -\frac{j^2}{\sigma} \tag{1}$$

Equation (1) is subjected to the following boundary conditions:

$$-\lambda\frac{dT}{dx}\Big|_{x=0} = q_0 \tag{2}$$

$$-\lambda\frac{dT}{dx}\Big|_{x=d} = \epsilon\sigma_{\rm SB}T_{\rm d}^4 \tag{3}$$

For the case of constant properties, the system of Eqs. (1-3) may be readily integrated to give

$$\epsilon\sigma_{SB}T_d^4 = \frac{j^2d}{\sigma} + q_0 \tag{4}$$

and

$$\lambda T_d = -\frac{j^2d^2}{2\sigma} - q_0d + \lambda T_0 \tag{5}$$

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*Assistant Professor. Member AIAA.

†Senior Scientist and Manager of Advanced Concepts. Member AIAA.

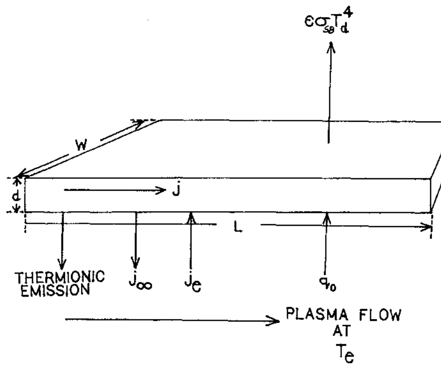

Fig. 1 The hollow cylindrical anode is shown in this idealization as a long thin slab of length L, width W, and thickness d.

Now, if q_0 is known and j is given, Eqs. (4) and (5) represent two equations for the two unknowns T_0 and T_d.

The heat flux on the anode inner surface q_0 must be found by considering particle bombardment from the plasma through the sheath. Consider, therefore, a power balance per unit area on the anode inner surface. The focus here is on conditions prior to sheath reversal; therefore, the anode is at a lower potential with respect to the plasma potential at the sheath edge. The power balance, then, mainly consists of a balance between electron bombardment and thermionic emission cooling. It can be shown that under these operating conditions evaporative cooling is a relatively small effect. Ion bombardment may be easily included in our analysis, but we neglect it for simplicity. Although this is expected to influence final results quantitatively, the conclusions regarding the mechanisms responsible for a thermal runaway will not be altered. We then have

$$q_0 = j_e(\phi_A + 2kT_e/e) - AT_0^2 \times (\phi_A + 2kT_0/e)\exp\{-e\phi_A/kT_0\} \tag{6}$$

The electron current density from the plasma j_e can be determined from overall current conservation

$$j_e = j_\infty + AT_0^2\exp\{-e\phi_A/kT_0\} \tag{7}$$

Inclusion of ion bombardment would involve additional terms on the right hand side of Eqs. (6) and (7). The anode sheath drop may be determined from Eq. (7) to be

$$V_A = -\frac{kT_e}{e}\ln[(j_\infty + j_E)/j_r] \tag{8}$$

where $j_r = en_e(kT_e/2\pi m_e)^{1/2}$ and $j_E = AT_0^2\exp\{-e\phi_A/kT_0\}$. The anode is at a potential of $-V_A$ with respect to the plasma-sheath edge (taken as the $V=0$ datum) in the regime under consideration. Combining Eqs. (6) and (7) gives

$$q_0 = j_\infty\left(\phi_A + \frac{2kT_e}{e}\right) + \frac{2AkT_0^2(T_e - T_0)}{e}\exp\{-e\phi_A/kT_0\} \tag{9}$$

Finally, combining Eqs. (4) and (5), we get an implicit equation for T_0

$$F(T_0) = \frac{j^2 d}{\sigma} + q_0 - \epsilon\sigma_{SB}\left(T_0 - \frac{q_0 d}{\lambda} - \frac{j^2 d^2}{2\lambda\sigma}\right)^4 = 0 \tag{10}$$

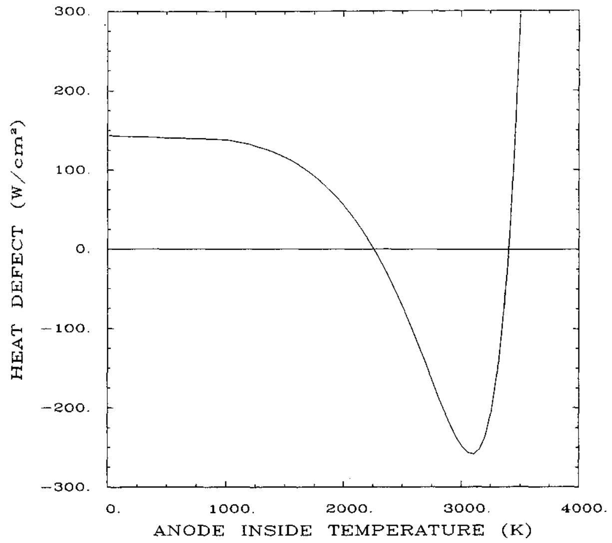

Fig. 2 A typical variation of the heat defect (defined as the net heat flux into the anode minus the net heat flux out) is shown here vs the anode inside surface temperature T_0.

where q_0 is given by Eq. (9). Given d, L, W, T_e, and J, the axial current density throught the anode ranges up to $j = J/Wd$, and the average current density through the plasma at the anode surface is $j_\infty = J/WL$. Although this simplification is unnecessary (since j and j_∞ can be treated as separate pointwise parameters), it allows the thermal steady state to be related directly to the total current (a more tangible quantity) rather than the current densities. With these quantities, Eq. (10) may be solved for T_0. Once T_0 is determined, T_d may be readily obtained from Eqs. (4) or (5). Equations (9) and (10) have been solved in this manner for representative conditions in the MPD thruster. These results are discussed in the following section.

III. Analysis and Results

In the previous section, it was shown that an energy balance on the anode at steady state yields an implicit equation for the anode inside the surface temperature T_0. This equation [Eq. (10)] is nonlinear in T_0 and must be solved numerically. This section will focus on the solution of this implicit equation, resulting in the discovery of a thermal runaway mode and culminating in a discussion of various operating limits.

For illustrative purposes, let us consider a tungsten anode of width 30 cm and length 10 cm. Let the electron temperature T_e be fixed at 15,000 K. The typical variation of the implicit function $F(T_0)$ vs T_0, given by Eq. (10), is displayed in Fig. 2. The function $F(T_0)$ represents the net heat into the anode per unit area per unit time. It is immediately evident from Fig. 2 that two steady-state solutions exist. Without performing a stability analysis, one can conclude on the basis of physical reasoning that the lower temperature is a stable root and the higher one is an unstable root. Consider the smaller temperature first. Any perturbation to the left of this root (i.e., on the lower temperature side) results in a net heat flux into the anode. To counter this, the surface temperature must increase in order to maintain a steady state. Similarly, any small perturbation to the right (i.e., on the higher temperature side) results in a net heat flux out of the anode. To counter this, the surface temperature must decrease. Thus, we see that the tendency of the thermal response is to drive the temperature toward this root. Using this physical argument on the second root (i.e., higher temperature), we may conclude that it is unstable. If the surface temperature increases just above the value given by the second root, the steady-state energy balance indicates a further increase in temperature in order to maintain a steady state. This is because the only possible way of losing energy is through radiation and by thermionic emission, both of which increase with increasing temperature. This results in a thermal runaway with eventual melting of the anode.

A physical interpretation of the two aforementioned solutions will now be discussed. Examination of Eq. (8) indicates that the anode sheath drop decreases as the net plasma current density increases. This results in increased electron bombardment, as can be seen from Eqs. (6) and (7), causing heating of the anode surface. Two possible scenarios may ensue. The resulting increase in the surface temperature leads to an increase in the emission current density, which further lowers the anode sheath voltage drop. This positive feedback may repeat itself until thermal runaway leads to local melting of the surface. The second scenario is stable, steady operation arising from the fact that sufficient cooling of the anode prevents the thermal runaway by excessive electron bombardment. The two solutions found to Eq. (10) represent these two situations.

The anode surface temperature T_0 for stable operation is found to be strongly dependent on the total current, anode geometry, and the anode work function. Figure 3 shows the anode thickness for various total currents, and Fig. 4 shows the anode inside surface temperature vs total current for various anode thicknesses. From these results, it is evident that for a given total current, there exists an optimum thickness for achieving a minimum anode temperature. An interesting feature of the pair of solutions found to the steady-state heat balance equation is that the stable and unstable roots start moving toward each other as the current is increased. This continues until a critical value of the current is reached, beyond which no steady solution can be found below the melting point of the material. Also, for a fixed total current, these roots approach each other as the thickness is increased. It is possible that the analytical behavior of this unstable thermal runaway point will yield insight into electrode material breakdown.

Some interesting limits may be readily obtained by analyzing the equations presented in Sec. II. For instance, from Eq. (5) it is clear that

$$j^2d^2/2\sigma < kT_0 \quad \text{or} \quad j < j_c = (2k\sigma T_0)^{1/2}/d \tag{11}$$

which is very similar (within a proportionality constant) to the thermal runaway condition obtained by Hantzsche[8] for cathode spots. However, it must be pointed out that Eq. (11) is obtained here for an anode of constant electrical conductivity, operating in the diffuse mode. This sharply differs from Hantzsche's consideration of a cathode spot with an electrical conductivity that varies according to the Wiedemann-Franz law (i.e., $\sigma \propto 1/T$). Furthermore, in constrast to Hantzsche's work, surface cooling due to thermionic emission *does not* prevent thermal runaway. This thermal runaway can occur under the steady, diffuse mode operation of the anode.

An important stability criterion may be derived by requiring that $\delta F/\delta T_0 < 0$ for stable diffuse operation. Using Eq. 7, we find

$$V_A \geq \frac{-kT_e}{e} \ell n \left[\frac{j_\infty + 0\left(\dfrac{4\epsilon J_{SB} T_d^3 T_0^2}{2\phi_A T_e}\right)}{j_r} \right] \tag{12}$$

For V_A below the value given by the right-hand side of Eq. (12), excessive electron bombardment will cause a thermal runaway. Thus, local surface melting can occur prior to sheath reversal.

A limit may also be found for the plasma current density j_∞. Considering thin anodes (i.e., $T \approx T_0$ throughout the anode), neglecting ohmic heating and any external cooling except for radiation, we may write the energy balance at steady state as

$$\epsilon\sigma_{SB}T_0^4 = j_\infty\left(\phi_A + \frac{2kT_e}{e}\right) + \frac{2kAT_0^2}{e}(T_e - T_0)\exp(-e\phi_A/kT_0) \tag{13}$$

Using the fact that $T_e \gg T_0$, differentiating with respect to T_0, and setting $\partial j_\infty/\partial T_0 = 0$, we obtain

$$(j_\infty)_{max} \approx \frac{[1-(4kT_{0c}/e\phi_A)]}{[\phi_A + (2kT_e/e)]}\epsilon\sigma_{SB}T_{0c}{}^4 \tag{14}$$

where $(j_\infty)_{max}$ is the extremum plasma current density, and T_{0c} is the critical value of T_0 where $\partial j_\infty/\partial T_0 = 0$. Additional heat transfer to an external coolant results only in a slight modification of Eq. (14):

$$(j_\infty)_{max} \approx \frac{[1-(4kT_{0c}/e\phi_A)]}{[\phi_A + (2kT_e/e)]}\epsilon\sigma_{SB}T_{0c}{}^4 + \frac{[1-(kT_{0c}/e\phi_A)]}{[\phi_A + (2kT_e/e)]}hT_{0c} \tag{15}$$

where h is the heat transfer coefficient between the anode (at T_0) and an external coolant (at $T_c \ll T_0$). For $T_e = 20{,}000$ K, Eq. (14) approximately yields 47.6 A/cm^2 for a tungsten anode ($\phi_A = 4.52$ V) and 5.4 A/cm^2 for a thoriated tungsten anode ($\phi_A = 2.63$ V). The lower work function yields a lower maximum plasma current density because of a lower value of T_{0c}. Since these limiting current densities are far lower than

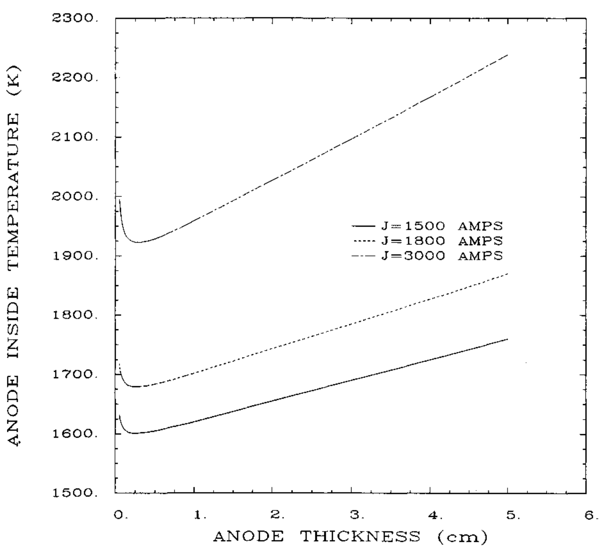

Fig. 3 The anode inside surface temperature is plotted here vs anode thickness, for various total currents.

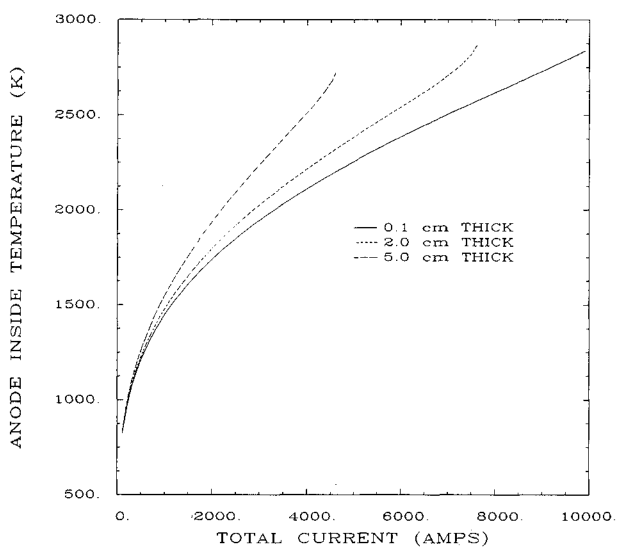

Fig. 4 The anode inside surface temperature is shown here vs total current, for various anode thicknesses.

those required for a steady MPD discharge, it is clear that the anode must be cooled externally. This is done in practice.[1,4]

IV. Summary and Conclusions

An anode energy balance has been performed that includes the effects of the anode sheath. Results show that under many conditions, two steady-state solutions can be found. One of these corresponds to a stable operating point, the other to a thermal runaway. The stable root that gives the anode inside surface temperature at steady state was found to be strongly dependent on the total current, anode geometry, and the material work function. A stability condition in the form of a minimum anode sheath voltage drop [Eq. (12)] has also been given.

Several conclusions can be derived from the anode thermal analysis:

1) There exists, for a given current density, an optimum thickness for achieving a minimum anode temperature.

2) Under many conditions, there exists a pair of steady-state solutions to the heat balance, one of which is a stable operating point, the other of which is an unstable thermal runaway point.

3) There exists a maximum steady operating current density for a given electrode geometry based on melting temperature and cooling considerations.

The thermal runaway mode discovered for the anode may explain anode spots under MPD conditions. Although some limits have been found, anode heat transfer effects need to be explored further in order to learn more about what controls the limits of stable operation. Since external cooling is seen to be extremely important, detailed experimental results quantifying cooling rates for various operating conditions and geometries would be valuable. Radiative heat transfer from the plasma, which has been neglected here, is another important area for further research.

Acknowledgments

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